LECTURE NO 32

Topics

- power and the pointing vector,
- reflection of a plain wave in a normal incidence

Power and Poynting vector

As mentioned before, energy can be transported from one point (where a transmitter is located) to another point (with a receiver) by means of EM waves. The rate of such energy transportation can be obtained from Maxwell's equations:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \tag{10.58a}$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \tag{10.58b}$$

Dotting both sides of eq. (10.58b) with **E** gives

$$\mathbf{E} \cdot (\nabla \times \mathbf{H}) = \sigma E^2 + \mathbf{E} \cdot \varepsilon \frac{\partial \mathbf{E}}{\varepsilon t} \tag{10.59}$$

But for any vector fields **A** and **B** (see Appendix A.10)

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}).$$

Applying this vector identity to eq. (10.59) (letting $\mathbf{A} = \mathbf{H}$ and $\mathbf{B} = \mathbf{E}$) gives

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}) + \nabla \cdot (\mathbf{H} \times \mathbf{E}) = \sigma E^2 + \mathbf{E} \cdot \varepsilon \frac{\partial \mathbf{E}}{\partial t} \tag{10.60}$$

From eq. (10.58a),

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}) = \mathbf{H} \cdot \left(-\mu \frac{\partial \mathbf{H}}{\partial t} \right) = -\frac{\mu}{2} \frac{\partial}{\partial t} (\mathbf{H} \cdot \mathbf{H}) \tag{10.61}$$

and thus eq. (10.60) becomes

$$-\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \nabla \cdot (\mathbf{E} \times \mathbf{H}) = \sigma E^2 + \frac{1}{2}\varepsilon\frac{\partial E^2}{\partial t}$$

Rearranging terms and taking the volume integral of both sides,

$$\int_v \nabla \cdot (\mathbf{E} \times \mathbf{H})\, dv = -\frac{\partial}{\partial t}\int_v \left[\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right] dv - \int_v \sigma E^2\, dv \qquad (10.62)$$

Applying the divergence theorem to the left-hand side gives

$$\oint_S (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S} = -\frac{\partial}{\partial t}\int_v \left[\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right] dv - \int_v \sigma E^2\, dv \qquad (10.63)$$

↓		↓		↓	
Total power leaving the volume	=	Rate of decrease in energy stored in electric and magnetic fields	−	Ohmic power dissipated	(10.64)